THE HYPERFINE STRUCTURE OF SOME SINGLY-IONIZED BISMUTH LINES

-by-

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This is to certify that the undersigned have read and recommend to the Committee on Graduate Studies for acceptance a thesis submitted by J. Stiles Beggs, B.Sc., entitled:

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CONTENTS

1. Introduction

Light, when emitted by atoms chemically uncombined and in the gaseous state, has the peculiarity of being concentrated in definite wave lengths which are characteristic of the element emitting it. According to the Quantum Theory a given kind of atom possesses only certain discrete energy levels characteristic of the kind, and when changing from higher to lower energy levels light is emitted whose wave length is inversely proportional to this energy difference. Hence each element has a characteristic spectrum.

When this light is passed through a narrow slit and dispersed, i.e. the various wave lengths passed in different directions, and allowed to fall on a photographic plate, images of the slit are formed which are commonly referred to as spectral lines, or merely as lines. It is found that these lines sometimes fall closely together in small groups such as the two well known yellow sodium lines. This is called Fine, or Multiple, Structure. (As a matter of fact in the heavy atoms the components of a group may be widely separated.) From what we have said above this Fine Structure is evidently due to small changes in wave length caused by small changes in energy differences. Thus there must be groups of energy levels in the atom lying very close together.

Lines which appear quite sharp in a spectrum obtained with a spectrograph of low dispersion, when examined by apparatus of greater dispersive power such as the Lummer Gehrcke plate used here, break up into a number of components. This is called Hyperfine Structure and is due to minute differences between energy levels of a group, attributed to the effect of the nucleus on the energy of the atom, or to isotopes.

2. The Theory of Hyperfine Structure.

Since there are no isotopes of bismuth its hyperfine structure is believed to be due to slight differences in the energy levels of a group caused by the rotation of the nucleus of the atom.

The usual notation is as follows,-

i = the nuclear spin quantum number, such that the mechanical moment of momentum of the nucleus = $ih/2\pi$.

j = the inner quantum number which takes the values $l + s,\ l + s - 1, \ldots\ldots\ldots |l - s|$.

l = the new azimuthal quantum number.

s = the electron spin quantum number .

f = the hyperfine structure quantum number .

f is found to be due to combinations of i and j in the same way that j is due to combinations of l and s .

Therefore, $f = i + j,\ i + j - 1, \ldots\ldots\ldots |i - j|$.

The number of hyperfine levels in a group is thus given by the smaller of $2j + 1$ or $2i + 1$ since there is a level for each value of f.

When energy changes occur f obeys the same selection rule as j, i.e.,

$\Delta f = \pm 1$ or 0 with $f = 0$ to $f = 0$ forbidden.

It will be seen that counting the number of components in a given spectral line and comparing with the theoretical number affords a check on the correctness of the j values assigned to the energy levels giving rise to that line. For example in the case of $\lambda 5209$ of Bi II classified by McLennan, McLay and Crawford [1] as $2^{\circ}_{1} - 9_{2}$,since $i = 9/2$

[1] Proc. Royal Society, Vol. 129, p 585 .

for Bi we have the following values for f,-

$i = 9/2$, $j = 1$, $f = 7/2, 9/2, 11/2$.

$i = 9/2$, $j = 2$, $f = 5/2, 7/2, 9/2, 11/2, 13/2$.

Applying the selection principle we obtain the following 9 components,-

f	5/2	7/2	9/2	11/2	13/2
7/2	X	X	X		
9/2		X	X	X	
11/2			X	X	X

In the results this line was found to have nine components as predicted.

In the same manner the line λ 4259 classified in the paper mentioned above as $8^{\circ}_{3} - 14_{4}$ should have 21 components according to the theory.

3. The Light Source and Method of Obtaining Photographs of Hyperfine Structure

The disposition of the apparatus is shown in Fig.2 ,

The source[1],S Fig.2 was Bi in a hollow cathode in an atmosphere of helium at a pressure of about .6 cm. of Hg . The arrangement is shown in Fig.1. The helium was circulated through the cathode by means of two mercury pumps in tandem and cleaned by passing through charcoal immersed in CO_2 snow in lieu of liquid air. The relatively high pressure of helium was used to keep the liquid Bi from diffusing out of the cathode. Outside of the aluminum anode was a glass tube of large diameter with a quartz window sealed on with wax at the end. The tube was cooled by being immersed in a large bath through which water constantly circulated. The water bathwas also provided with a quartz window to let through the ultra-violet light.

A hollow carbon cathode was tried at first but gave trouble due to absorbed gas which continued to pour out after thirty six hours of steady pumping. A one piece mol ybdenum cathode was then tried which was very satisfactory since it cleaned up in a few minutes and could be operated at red heat without trouble.

The discharge was found to operate most satisfactorily with about 900 ohms in series with a D. C. generator at a terminal voltage of 1200. This gave a large starting voltage and yet prevented arcing when the tube became hot. The normal operating current was about .4 amperes.

[1] Paschen, Ann. d. Physik 50, p901 (1916)

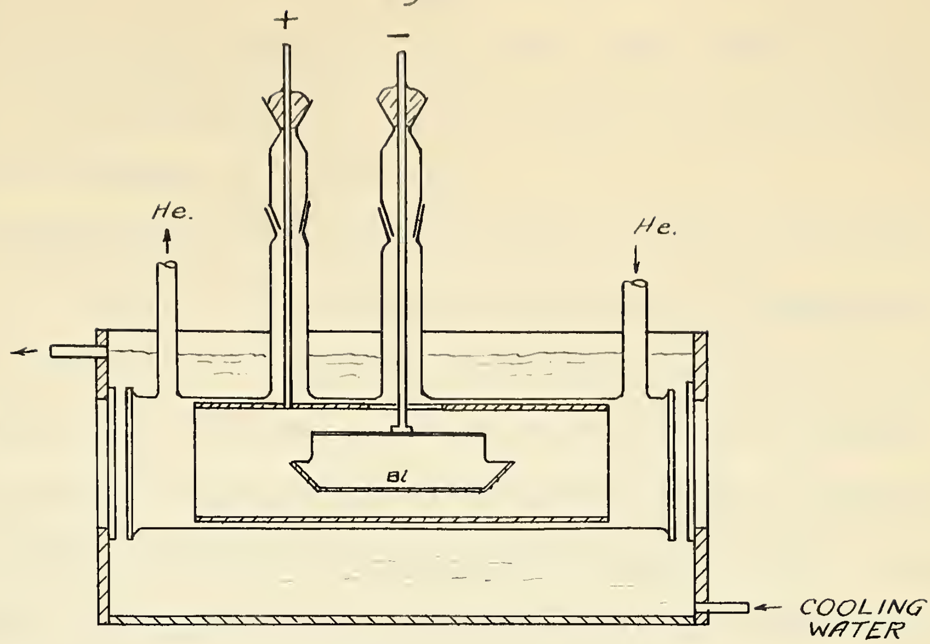

Fig.1 ,

Since quartz is doubly refracting, the Lummer-Gehrcke Plate is made with the optical axis in the face of the plate and perpendicular to the long edge. To cut out the ~~extra-~~ ordinary ray the light is polarized by placing a nicol N , Fig.2 with its short diagonal parallel to the face of the plate.

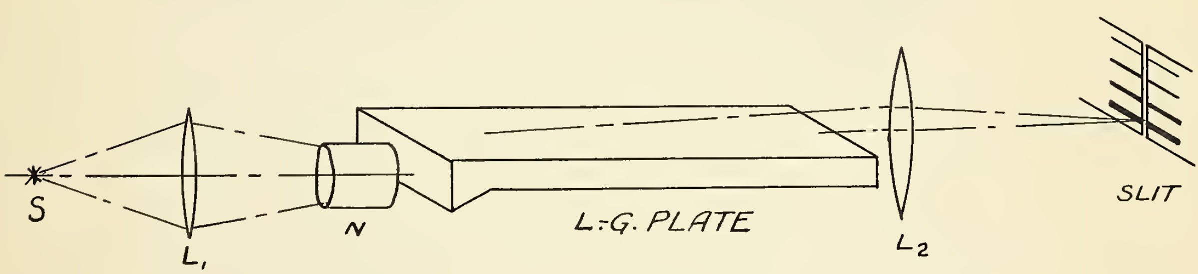

Fig. 2.

The light emerging from the plate is parallel and was focused by the lens L_2 on the slit of a Hilger constant deviation spectroscope with camera attachment. The interference pattern was clearly visible on the slit jaws. The lens L_2 had a focal length of 15 cms. The focusing was done photographically. A slit width of about 2 mm. was used. An

exposure of two and a quarter hours was found sufficient to bring out the satellites. The lens L_1 was used to throw an image of the cathode on the nicol.

A microphotometer was built similar to that described by Dr. C. S. Beals [1]. A few simplifications and changes were made. The compensating beam was found unnecessary and higher amplification was obtained by using 45 volts on the photo-electric cell and only 22 1/2 volts on the plates of UX 230 Radiotron radio valves. By keeping the batteries well charged and allowing the apparatus to run half an hour before using no serious trouble was encountered due to drift of the zero point.

[1] Royal Astronomical Society Vol. 92, p 196.

4. The Computation of the Wave Length Separations from the Interference Fringes Produced by the Lummer Gehrcke Plate.

In Fig.1 is shown a side view of the interferometer with typical rays passing through it.

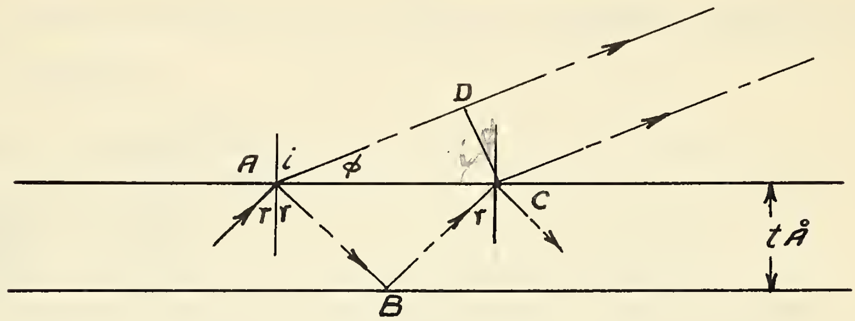

Fig.1 .

From the figure we see that the retardation q between AD and ABC is

AD = AC sin

AC = 2t tan

$$q = AB + BC - AD$$

$$= 2\,(\mu t \sec r) - 2\,(t \tan r) \sin i$$

$$= 2t\,(\mu / \cos r - \mu \sin^2 r / \cos r)$$ or

since $\mu \equiv \sin i / \sin r$

$$= 2t\mu \cos r$$

$1 - \sin^2 r = \cos^2 r$

$$= 2t\sqrt{\mu^2 - \sin^2 i}$$ by Fresnel's law of refraction

The condition for bright fringes is that $q = n\lambda$ where n is a positive integer.

$$n\lambda = 2t\sqrt{\mu^2 - \sin^2 i} \;\; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Squaring (1) and differentiating with λ constant,

$$n\lambda^2 \delta n = -\cancel{2}\,4\, t^2 \sin i \cos i \,\delta i$$

$$\delta i = \frac{-n\lambda^2}{\cancel{4}\, t^2 \sin 2i} \cdot \delta n \;\; \ldots\ldots\ldots\ldots\ldots (2)$$

λ constant.

or by (1)

$$\delta i = \frac{-\lambda\sqrt{\mu^2 - \sin^2 i}}{t \sin 2i}\, \delta n \;\; \ldots\ldots\ldots\ldots\ldots (3)$$

λ constant.

$= 2t\mu\sqrt{1 - \sin^2 r}$

Therefore the angular separation δi_1 between two consecutive orders of a given wave length is obtained from either (2) or (3) by putting $\delta n = 1$,

$$\delta i_1 = -\delta\phi_1 = \frac{-n\lambda^2}{2t^2 \sin 2i} \quad \ldots\ldots\ldots\ldots\ldots \quad (4).$$

λconstant.

For a given value of i, $\delta\phi_1$ will change with the wave length as follows,

$$\delta(\delta\phi_1) = \frac{n\lambda\,\delta\lambda}{t^2 \sin 2\phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5).$$

$$\therefore \frac{\delta(\delta\phi_1)}{\delta\phi_1} = \frac{2\,\delta\lambda}{\lambda}$$

In hyperfine structure work where $\delta\lambda/\lambda$ is very small the curves ($\delta\phi_1$, ϕ) for various components will thus be indistinguishable. This is an extremely important property, since it is sometimes very difficult to pick out the successive orders of a given component. We therefore start by plotting the ($\delta\phi_1$, ϕ) curve for a given strong component, the successive orders of which can be picked out with certainty. We then start to plot the ($\delta\phi_1$, ϕ) for a doubtful component and unless we make a mistake in choosing the successive orders the two curves will coincide. We thus have a valuable check on our work before any time is spent analyzing the patterns. The curve ($\delta\phi_1$, ϕ) for a given wave length is not of course continuous, but in practice one may draw a smooth curve through the points with considerable accuracy and the points for the orders of other wave lengths will lie on this continuous curve. Anticipating, I may say that the points for the 'ghosts' also lie on this same curve.

Squaring (1) and differentiating partially with respect to λ,

$$n^2 \lambda = 2\, t^2 \left(2\mu \frac{\partial \mu}{\partial \lambda} - \sin 2i \, \frac{\partial i}{\partial \lambda} \right)$$

$$\delta i = \frac{4\, t^2 \mu \frac{\partial \mu}{\partial \lambda} - n^2 \lambda}{2\, t^2 \sin 2i}\, \delta\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots (7).$$

n constant.

Consider now two wave lengths λ and λ' the latter being slightly the longer. Suppose that the difference in wave lengths is such that for a given interferometer plate and near grazing emergence a fringe of wave length λ' and order n coincides with a fringe of wave length λ and order $n+1$. Let us denote this wave length difference by $\delta\lambda_1$. Placing this value of $\delta\lambda$ in equation (7) we must get the same value for δi as we obtained from equation (4) for successive orders. Thus equating the right hand sides of equations (4) and (7) we get,

$$\delta\lambda_1 = \frac{n\lambda^2}{n^2\lambda - 4\, t^2 \mu \frac{\partial \mu}{\partial \lambda}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8).$$

$$= \frac{\lambda^2 \sqrt{\mu^2 - 1}}{2\, t \left(\mu^2 - 1 - \lambda \frac{\partial \mu}{\partial \lambda} \mu\right)} \quad \ldots\ldots\ldots\ldots\ldots (9).$$

by substituting for n from (1) and since $i \doteq \pi/2$ near grazing emergence.

The values of μ and $\partial\mu/\partial\lambda$ are obtained from equations (10) and (11) to follow.

Put $\nu = \frac{1}{\lambda}$ and $\frac{\partial \lambda}{\lambda^2} = -\partial \nu$

$$\Delta \nu = \frac{-\sqrt{\mu^2 - 1}}{2t\left(\mu^2 - 1 - \mu \frac{\partial \mu}{\partial \lambda}\right.}$$

Drude's Equation for the Index of Refraction of the Ordinary Ray in Quartz.

$$\mu_\lambda^2 = b^2 + \sum_1^3 \frac{M_k}{\lambda^2 - \lambda_k^2} \quad \dots\dots\dots\dots\dots\dots \quad (10).$$

where

μ_λ = the refractive index for the wave length λ.

λ = the wave length in Angstroms.

The constants are as follows,-

$b^2 = 4.5800$

$M_1 = 1.06 \times 10^6$	$\lambda_1^2 = 106 \times 10^4$
$M_2 = 4.4224 \times 10^9$	$\lambda_2^2 = 782,200 \times 10^4$
$M_3 = 7.1355 \times 10^{10}$	$\lambda_3^2 = 4,305,600 \times 10^4$

This equation is very accurate and checks to five figures with Kaye and Laby's Tables, page 76, 6th edition.

Differentiating (10) we have,

$$\frac{d\mu}{d\lambda} = -\frac{\lambda}{\mu} \sum_1^3 \frac{M_k}{(\lambda^2 - \lambda_k^2)^2} \quad \dots\dots\dots\dots\dots\dots \quad (11).$$

For extra ordinary use $\mu^2 = b^2 + \frac{[illegible]}{[illegible]}$

see Bailey [illegible] I [illegible] 75

use modern values for λ.

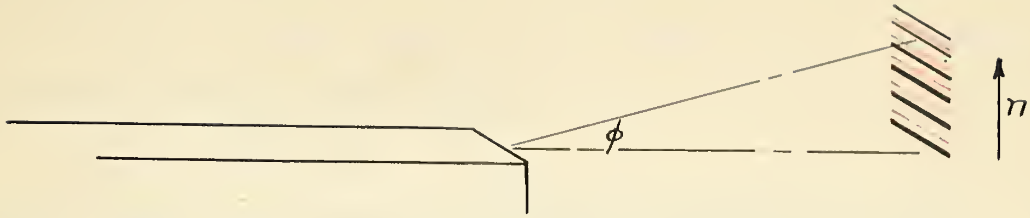

Fig.2 .

From equation (1) we see that for a given λ, n increases with ϕ, and from (1) and (10) that for a given order the longer wave length lies above the shorter.

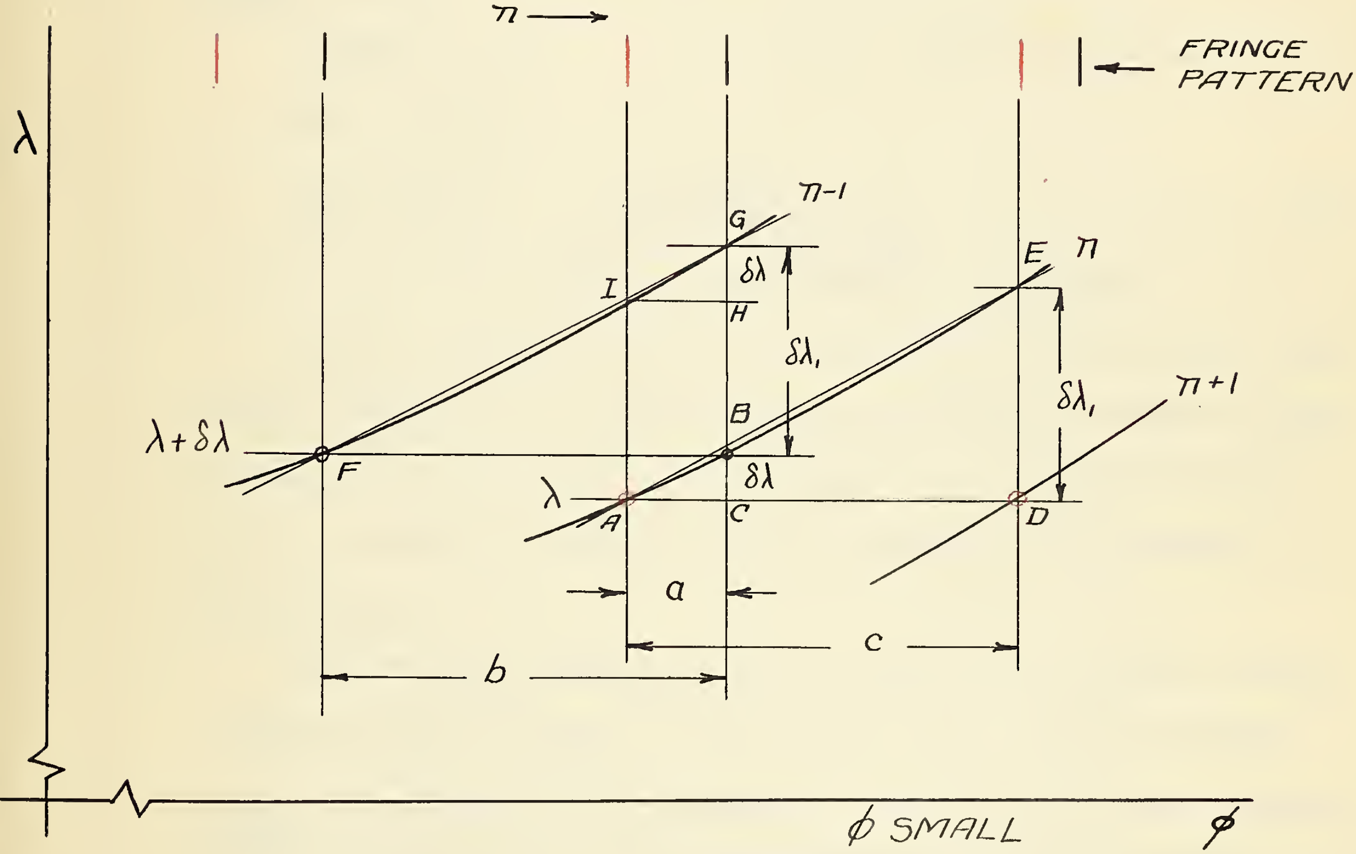

Fig.3 .

For small angles ϕ is proportional to the distance of the fringe from the line of intersection between the plane containing the top of the Lummer plate and the photographic plate as shown in Fig.2 .

In Fig.3 equation (1) is plotted (i replaced by ϕ) for three consecutive values of n. This figure shows clearly

the meaning of $\delta\lambda_1$ as the increase in wave length necessary for a fringe of wave length $\lambda + \delta\lambda_1$ and order n to fall on top of a fringe of wave length λ and order n + 1 . Comparing the similar triangles ABC , AED ,

$$a / c = BC / \delta\lambda_1$$

also, since the curves are concave upwards, BC > $\delta\lambda$

$$\therefore a / c > \delta\lambda / \delta\lambda_1$$

In the similar triangles GFB, BAC ,

$$HI / b = \delta\lambda / \delta\lambda_1$$

also a < HI

$$\delta\lambda / \delta\lambda_1 > a / b \qquad \therefore \; a / c > \delta\lambda / \delta\lambda_1 > a / b$$

$$\therefore \delta\lambda \doteq \frac{2a}{b+c}\,\delta\lambda_1$$

$$\delta\lambda \doteq \frac{2a}{b+c} \cdot \frac{\lambda^2\sqrt{\mu^2-1}}{2t\,(\mu^2-1-\lambda\mu\,\partial\mu/\partial\lambda)} \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (12)$$

for patterns near grazing emergence.

It is evident that (12) is not sufficient for the complete analysis of a pattern since the orders of the various fringes are not known nor the degree of overlapping which may be present. To carry out the analysis the procedure is as follows,-

The fringes produced by two plates of different thickness are photographed in succession, the coarse structure being dispersed in a horizontal direction by a prism spectrograph and the hyperfine structure vertically by means of the interferometer . We thus obtain on the photographic plate wide images of the slit corresponding to the spectral lines, which are crossed by the horizontal fringes of the hyperfine structure. The photographic plate is then placed in the microphotometer with the fringes parallel to the microphotometer slit and moved across the slit in steps of .001 cm. the

the intensity of the fringe being measured at each step, This movement is proportional to ϕ and in the direction of increasing ϕ. The intensity of the fringe is then plotted against the distance along the pattern, i.e. against ϕ, as in Fig. 4. Each peak in the intensity curve is a fringe in the pattern.

A $(\delta\phi_1, \phi)$ curve is then plotted as described on page 8 to make sure of the successive orders of any doubtful fringes

Different orders of the same wave length are arbitrarily named in succession as a_1 , a_2, a_3, etc. for the thin plate and a'_1, a'_2, ...b'_1, b'_2 ... etc. for the thick plate. Of course it would be only a coincidence if a and a' should happen to be the same hyperfine component.

Referring to Fig, 4 if a_1 and b_1 are of the same order, then by equation (1) b is of greater wave length than a, but if b_1 and a_2 are of the same order, a is of greater wave length than b. Since it is not known which is the case both wave length separations are computed by equation (12) and placed in a table or matrix, Fig.6 . a - b is placed in column a and row b, while b - a is placed in column b and row a. Since a cannot be both $\gtrless$ b, half the numbers in the matrix must be false. A similar table ,Fig.7 , is constructed for the other plate.

The two tables are now compared, the object being to find equal separations. For instance .103 appears in both tables. Since the naming of the components was arbitrary the primed components may be renamed tentatively by calling b',c and a',a , and placing these new names in the outer row and column. Thus a - c = .103 in both tables, and c - a may be crossed off as false. (In this simple case there would be

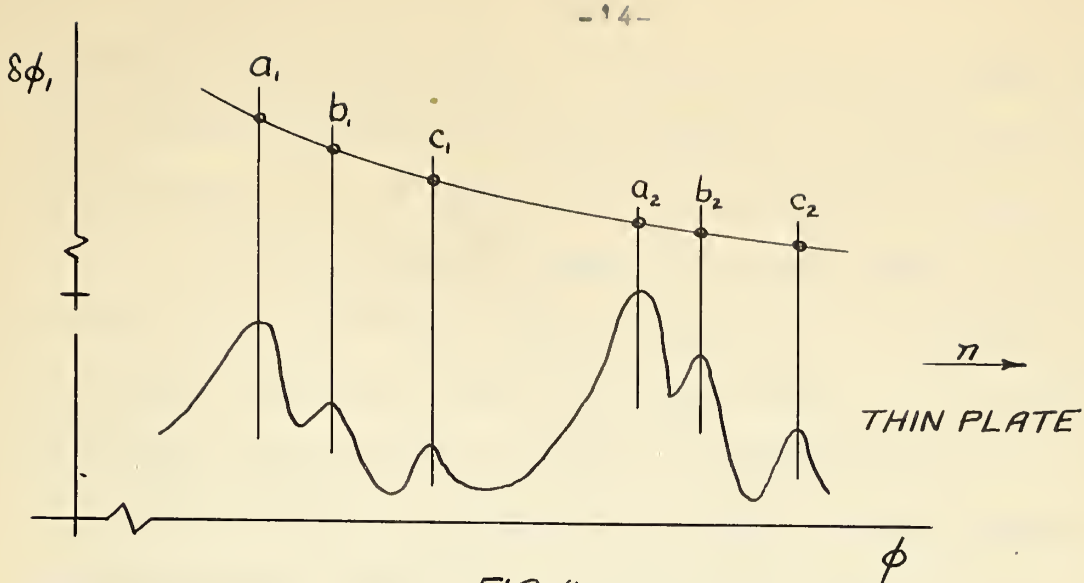

FIG. 4

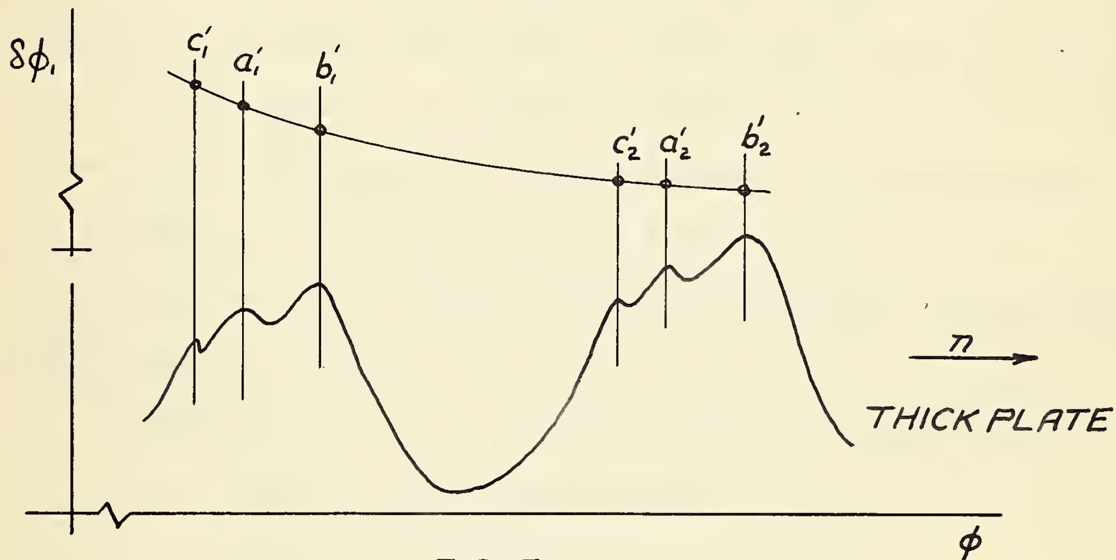

FIG. 5

$\delta\lambda_1 = .162$ THIN PLATE

	a	b	c
a	0	.026	.059
b	.134	0	.028
c	.103	.130	0

FIG. 6

$\delta\lambda_1' = .125$ THICKPLATE

		a	c	b
		a'	b'	c'
a	a'	0	.022	.117
c	b'	.103	0	.096
b	c'	.008 .133	.029	0

FIG. 7

no choice but to identify c' with b, but in an actual case there would be many more lines present.) It might next be noticed that b - c = .130 and this difference sought for in row c of Fig.7, since it does not appear there the conclus- is that $b \not> c$. Therefore the difference c - b = .028 is sought in column c of Fig.7 and it is found that c - c'= .029. This difference might easily be due to experimental error ,so that it is justifiable to rename b',c . There is now complete correspondence between the tables except that a - b = .134 for the thin plate, and .008 for the thick. Now the .008 for a'-c' is based on the assumption that a_2' and c_2' are of the same order. If, however a_2' and c_1' are of the same order,

$$a'-c' = .008 + \delta\lambda_1 = .008 + .125 = .133$$

which agrees with the wave length difference .134 obtained from the thin plate.

Therefore $a > c > b$ and the separations are known.

As a check on the computations,

$$(a-c) + (c-b) = (a-b)$$

$$.103 + .0285 \stackrel{?}{=} .1335$$

It may be noticed that, e.g. ,

$$(a-b) + (b-a) \doteq \delta\lambda_1$$

Thus for a first trial only one of a-b or b-a need be computed from the graph.

The following should be kept in mind when analysing the interference patterns,-

If the matrix for a given Lummer-Gehrcke plate has been set up correctly, then it will be self consistent, i.e. if we suppose that $a > b > c$ then

$$(a-b) + (b-c) = (a-c) \quad \ldots\ldots\ldots\ldots (33)$$

even if the statement $a > b > c$ is false and hence this con-

sistency is a check only on the measurements and computations

Also if a'-b' = a - b where the primes refer to a second plate, and as a consequence a is identified with a' and b with b', the statement b - a is automatically ruled out as false. Suppose also b'- c' is found to equal b - c and therefore c and c' are identified. This increases the probability that the identification of the first two pairs was correct. If the measurements and computations are accurate it must follow by (13) that a'- c' = a - c, and hence the latter is not a check on the identification of the fringes of one plate with those of another of different thickness.

When analysing the patterns of λ5209 with the help of an analysis made by Fisher and Goudsmit[1] it was noticed that there were two complete sets of lines corresponding to the ones listed by these experimenters. On further investigation it was found that each fringe of a given order and produced by light of a given wave length was accompanied by at least four 'ghosts', symmetrically placed on each side as shown in Fig.8. The subscripts denote the relative order of interference and the superscript the position of the ghost. The latter appear dotted in the diagram.

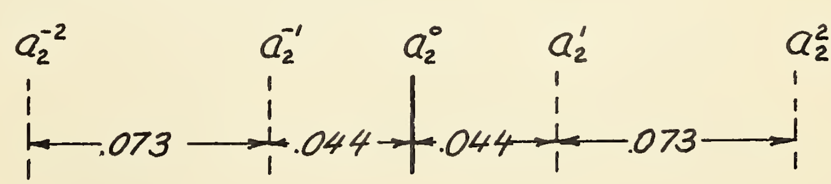

Fig.8.

The separations are in A and indicate the wave length separation from the true central fringe which a fringe of different wave length would have to possess in order to coincide with the ghost.

1 Phys. Rev. vol.37 p1060.

The first thing of interest to be noted is the symmetry. Secondly, that the displacement in Å was the same for both the thick and thin plates. It is to be noted however that they are of the same length although it is hard to see how this could account for the two plates giving the same separations.

Thirdly, the separations were nearly the same (.903 Å smaller) for λ4259 ,i.e. the effect is almost independent of the wave length.

Fourthly, the ratio of the successive separations is almost exactly 3:5 . This suggests a non-classical effect.

As far as can be ascertained there is no explanation of these ghosts in the literature.

5. Results

The results are given on the following pages. The agreement between the measurements from the two plates is fairly good as is the consistency of the measurements of any one plate.The measurements as given by Fisher and Goudsmit[1] of λ5209 as obtained by a grating are given for comparison.The grating failed to resolve the components b and c,apart from these the agreement is excellent.

It would appear that the Lummer-Gehrcke plate is an excellent interferometer for use with sources of moderate strength. The analysis of the interference patterns is long but the results are accurate. The main difficulty is due to 'ghosts' which make the patterns very crowded,they may however be eliminated with certainty from the results by means of a systematic analysis as outlined above. In some cases ghosts actually increase the accuracy of the measurements as a set of wave length differences can be calculated for each set of ghosts.

$\lambda 5209$ Å

The separations are in Å.

	a	b	c	d	e	f	g	h
b	.182 .179							
c	.188 x .187	.008 .008	Fisher & Goudsmit thick plate thin plate					
d	.401 .401 .401	.218 .221 .221	.213 .212 .213					
e	.450 .453 .454	.274 .274	.266 .266	.050 .051				
f	.587 .584 .585	.404 .406	.398 .398	.182 .183	.132 .131			
g	.776 .775 .776	.594 .595	.588 .588	.375 .376	.323 .321	x .192		
h	.912 .914 .915	.731 .734	.723 .727	.510 .511	.463 .461	.322 .324	.131 .133	
i	1.060 1.058 1.059	.876 .878	.872 .870	.658 .656	.606 .603	.474 .471	.283 .280	.143 .143

x = separation same as $\delta\lambda$, and therefore not resolved.

λ4259 Å

The separations are in Å.

	a	b	c
b	.094 .093	thin plate thick plate	
c	.198 .196	.103 .103	
d	.227 .225	.134 .133	.028 .029

6. Acknowledgments

I wish to take this opportunity of expressing my gratitude to Prof. Stanley Smith, whose guidance and help made this work possible.

Also I wish to express my appreciation of the kindly interest taken by the other members of the Physics Department in this work, especially Dr. Lang who supervised the construction of part of the microphotometer and Mr. Gleave in charge of apparatus.

One of the Lummer-Gehrcke plates used in this work is the property of the National Research Council of Canada to whom I am indebted for its use. Dr. Gowan very kindly loaned us a photoelectric cell for use in the microphotometer.